LIFETIME OF A NARROW CHANNEL IN A LIQUID

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Lifetime determination is considered for a narrow channel in a liquid collapsing in response to gravitational and capillary forces. Working formulas are derived to relate the lifetime to the properties of the liquid and the channel parameters. The calculations are compared with experiments on the effects of a focused high-power electron beam acting on a liquid and solid.

Concentrated heat sources such as electron and laser beams act on condensed media to produce channels a fraction of a millimeter in diameter; the channel walls are usually liquid, being kept in equilibrium by the pressure of the vapor generated by the beam. A narrow channel can exist in dynamic equilibrium for a certain time, but it begins to collapse when the heat source is removed.

An instance of an interesting collapse problem relates to the formation of deep melted zones in metals exposed to electron beams [1, 2]. Similar problems arise for focused laser beams acting on condensed materials, and also when one simulates such effect with small gas jets acting on liquids. High-speed cinematography has been applied [3] to the formation of narrow cavities in VKZh-94 oil with pulsed and continuous electron beams. Rayleigh [4] discussed the collapse of a spherical cavity suddenly formed in a liquid. It would seem that no detailed study has previously been made of the collapse of a narrow channel.

Here we consider the hydrodynamic problem of the motion of an incompressible liquid when a narrow channel collapses; the liquid may be a molten metal, for example. Here we do not discuss the channel formation mechanism.

The problem is formulated as follows. We have a cylindrical volume of incompressible liquid V, in which a cylindrical channel of radius R_0 and depth H_0 suddenly appears. We have to determine the time taken to fill the channel with liquid.

The following are the Navier–Stokes equations, and also the equations for continuity and conservation of matter in a cylindrical coordinate system with axial symmetry (independence of angle φ):

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=K_r-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_r}{\partial r}\right)+\frac{\partial^2 v_r}{\partial z^2}\right] \quad (1)$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)=K_z-\frac{\partial P}{\partial z}+\mu\left[\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right] \quad (2)$$

$$\frac{\partial v_z}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0 \quad (3)$$

$$\iiint_V dV=V_0=\text{const} \quad (4)$$

Here ρ is density, while v_r and v_z are the radial and axial components of the liquid velocity correspondingly, with K_r and K_z the radial and axial components of the volt force acting on the liquid and μ the dynamic viscosity.

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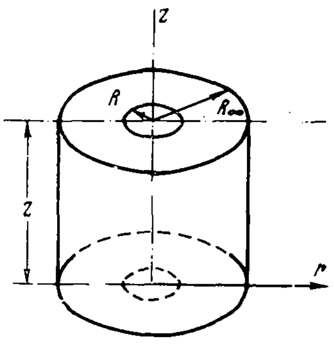

Fig. 1

We make some simplifying assumptions and consider the solutions to particular problems for special cases. The approximations are that first of all we specify the upper and lower boundaries of the liquid (channel compression), with reference to the case of a fixed lateral boundary (liquid sinking) and flow into the channel (the radial motion of the liquid is neglected).

We consider the case where the upper and lower boundaries (Fig. 1) are fixed, i.e., the volume is redistributed by shift in the side boundaries $R_i = R_i(t)$ $(i = 1, 2)$; the quantities dependent on z then disappear from the equations and (1)-(3) takes the form

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r}\right) - \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_r}{\partial r}\right)\right] = -\frac{\partial P}{\partial r} \tag{5}$$

$$\partial r v_r / \partial r = 0 \tag{6}$$

The boundary and initial conditions are put in the form

$$\tau_{rr} = -P + 2\mu\,\partial v_r / \partial r = \pm\sigma_i / R_i \quad \text{for} \quad r = R_i(t) \tag{7}$$

Here the plus sign refers to i = 1 and the minus to i = 2;

$$R_2^2(t) - R_1^2(t) = R^{*2} - R_0^2, \; R^* = R_2(0), \; R_0 = R_1(0)$$
$$v_r = 0 \quad \text{for} \quad t = 0 \tag{8}$$

From Eq. (6) we get $v_r = r^{-1}f(t)$, where r is the current coordinate $[R_1(t) \le r \le R_2(t)]$; we integrate Eq. (5) with respect to r from R_1 to R_2 to get

$$\ln\frac{R_2(t)}{R_1(t)}\frac{\partial f(t)}{\partial t} + \left(2\nu f - \frac{f^2}{2}\right)\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) + \frac{\sigma}{\rho}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0 \tag{9}$$

We convert to new dimensionless variables via the formula

$$a = \left(\frac{R^{*2}}{R_0^2} - 1\right), \quad b = \frac{\sigma R_0}{\rho\nu^2}, \quad \vartheta = \frac{\nu t}{R_0^2}, \quad \xi = \frac{R^2(t)}{R_0^2}, \quad \psi(t) = \frac{f(t)}{\nu}$$

where $\nu = \mu / \rho$, and transform Eqs. (7)-(9) to

$$\ln\left(1 + \frac{a}{\xi}\right)\frac{d\psi}{d\vartheta} + \frac{\psi(4 - \psi)a}{\xi(\xi + a)} + 2b\left(\frac{1}{\sqrt{\xi}} + \frac{1}{\sqrt{\xi + a}}\right) = 0$$
$$d\xi / d\vartheta = 2\psi, \quad \psi(0) = 0, \quad \xi(0) = 1 \tag{10}$$

The collapse time is found from

$$\vartheta = \frac{1}{2}\int_1^0 \frac{d\xi}{\psi(\xi)} \tag{11}$$

where $\psi(\xi)$ is determined from the solution to

$$2\psi\ln\left(1 + \frac{a}{\xi}\right)\frac{d\psi}{d\xi} + \frac{\psi(4 - \psi)a}{\xi(\xi + a)} + 2b\left(\frac{1}{\sqrt{\xi}} + \frac{1}{\sqrt{\xi + a}}\right) = 0$$
$$\psi = 0 \quad \text{for} \quad \xi = 1 \tag{12}$$

The solution to Eq. (12) cannot be found in general form, so we consider some particular cases; a solution has been given [5] for b = 0. We neglect the viscosity and get from Eqs. (11) and (12) after transformation that

$$\tau = \frac{1}{4}\sqrt{\rho R_0^3 / \sigma}\int_0^1\left[\frac{\ln(1 + a/\xi)}{1 + \sqrt{a + 1} - \sqrt{\xi} - \sqrt{a + \xi}}\right]^{1/2} d\xi \tag{13}$$

If $\xi \ll a$ (narrow channel), we have

$$\tau = \sqrt{\rho R_0^3/\sigma}\int_0^1\left[\frac{\ln R^*/R_0 - \ln x}{1-x}\right]^{1/2} x\,dx \tag{14}$$

The interval of Eq. (14) converges, and the value lies in the range

$$\frac{2\sqrt{2}}{3}\sqrt{\rho R_0^3/\sigma}\ln\frac{R^*}{R_0} < \tau < \frac{4}{3}\sqrt{\rho R_0^3/\sigma}\left(\ln\frac{R^*}{R_0} + 0.283\right) \tag{15}$$

Then the collapse time can be estimated from

$$\tau \approx \sqrt{\rho R_0^3/\sigma}\ln R^*/R_0 \tag{16}$$

We insert the numerical values $R_0 = 10^{-2}$ cm, $\rho = 10$ g/cm^3, $R^* = 1$ cm, $\sigma = 10^3$ dyne/cm characteristic of narrow channels in molten metals to get $\tau \approx 0.5$ msec.

We now consider the case where the side boundary is fixed, while the volume redistribution occurs by two-dimensional displacement of the upper boundary, i.e., we assume that $v_z(r, z, t) = v_z(z, t)$; from Eq. (4) we have

$$(R^{*2} - R^2)z = (R^{*2} - R_0^2)z_0 \tag{17}$$

We differentiate Eq. (17) with respect to t to get

$$2R\frac{dR}{dt}z = (R^{*2} - R^2)\frac{dz}{dt} \tag{18}$$

or

$$\frac{dz}{dt} = v_z(z, t) = \frac{2Rzv_r(R)}{R^{*2} - R^2}, \quad \frac{\partial v_z}{\partial z} = \frac{2Rv_r(R)}{R^{*2} - R^2}$$

We substitute Eq. (18) into (3) with $v_r = 0$ for $r = R^*$ to get

$$v_r(r) = -f(t)(R^{*2} - r^2)/r \tag{19}$$

We integrate Eq. (1) with respect to r from R to R* and Eq. (2) with respect to z from 0 to h using Eqs. (18) and (19) to get a system of ordinary equations:

$$\begin{gathered}\frac{df}{dt}\left[R^{*2}\ln\frac{R^*}{R} - \frac{1}{2}(R^{*2} - R^2)\right] + \frac{1}{2}f^2(R^{*2} - R^2)^2/R^2 = \frac{1}{\rho}[P(R^*) - P(R)] \\ \sigma_{rr} = -p + 2\mu\,\partial v_r/\partial r \quad \text{for} \quad r = R^* \text{ and } R \\ dR/dt = -f(t)(R^{*2} - R^2)/R, \quad f(0) = 0\end{gathered} \tag{20}$$

$$\begin{gathered}\frac{df}{dt} - 2f^2 = \frac{g}{h} + \frac{1}{\rho h^2}[P(h) - P(0)] \\ \sigma_{zz} = -P + 2\mu\,\partial v_z/\partial z = 0 \quad \text{for} \quad z = 0,\ h \\ dh/dt = v_z|_{z=h} = -2f(t)h, \qquad f(0) = 0\end{gathered} \tag{21}$$

Systems (20) and (21) are independent because we have assumed that $v_z(r, z, t) = v_z(z, t)$, which means that v_r and $\partial v_z/\partial_z$ are independent of z; physically this means that the upper boundary can descend as a plane only if the forces acting on the liquid in the two mutually perpendiculars r and z are independent. In fact, v_r and v_z are coupled only via the equation of continuity (3) and the analogous law of conservation of matter (4).

This case splits up into two: a) system (20) corresponds to channel filling via capillary forces in the absence of gravity; b) system (21) corresponds to filling by descent of the liquid in the absence of capillary forces.

Consider case a; we have from (20) that

$$\begin{gathered}-\frac{df^2}{dR}\left[R^{*2}\ln\frac{R^*}{R} - \frac{1}{2}(R^{*2} - R^2)\right] + f^2\frac{R^{*2} - R^2}{R} - \frac{4\nu}{R}f = \frac{\sigma}{\rho R^*(R^* - R)} \\ f(R_0) = 0\end{gathered} \tag{22}$$

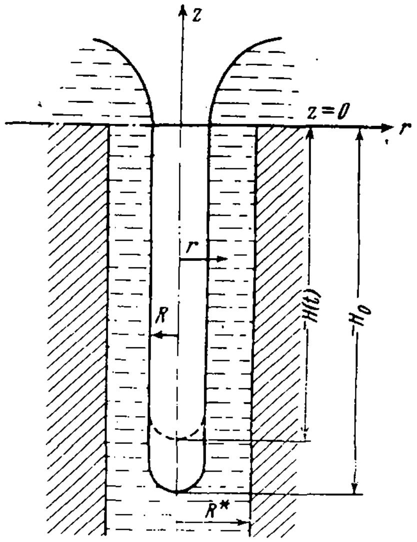

Fig. 2

The filling time is found from

$$\tau = \int_0^{R_0} \frac{R\,dR}{(R^{*2} - R^2) f(R)} \tag{23}$$

where $f(R)$ is the solution to Eq. (22); if $\nu = 0$

$$f^2(R) = \frac{\sigma}{\rho R^*} \ln \frac{R^* - R}{R^* - R_0} \Big/ \left[R^{*2} \ln \frac{R^*}{R} - \frac{1}{2}(R^{*2} - R^2) \right]$$

The filling time for narrow channels ($R^2 \ll R^{*2}$) is found in explicit form:

$$\tau_\sigma = \tau_0 \int_0^1 \left[\frac{B - \ln x}{1 - x} \right]^{1/2} x\,dx$$

$$\tau_0 = \left[\frac{\rho R_0^3}{\sigma} \left(1 - \frac{R_0}{R^*}\right) \right]^{1/2}, \qquad B = \ln \frac{R^*}{R_0} - \frac{1}{2} \tag{24}$$

Equation (24) is analogous to Eq. (14), so

$${}^4/_3 B\tau_0 < \tau_\sigma < \tau_0 {}^4/_3 \sqrt{2}\,(B + {}^5/_3 - 2\ln 2) \tag{25}$$

We see from Eqs. (15) and (25) that τ for the first case is of the same order as for case *a*, being dependent on the parameter $(\rho R_0^3/\sigma)^{1/2} \ln (R^*/R_0)$.

We have for case b from Eq. (21) that

$$\frac{df^2}{dh} + \frac{2}{h} f^2 = -\frac{g}{h^2} \quad \text{for} \quad f(h)|_{h=h_0} = 0 \tag{26}$$

The filling time is given as follows by analogy with Eq. (23):

$$\tau_g = \int_{h_0}^{h(0)} \frac{dh}{-2f(h)\,h}, \qquad h(R) = h_0 \frac{R^{*2} - R_0^2}{R^{*2} - R^2} \tag{27}$$

The solution to Eqs. (26) and (27) is

$$f(h) = \frac{[g(h_0 - h)]^{1/2}}{h}, \qquad \tau_g = \frac{R_0}{R^*} \sqrt{\frac{h_0}{g}} \tag{28}$$

We substitute the values $R_0 = 10^{-2}$ cm, $R^* = 1$ cm, $h_0 = 1$ cm into Eq. (28) to get $\tau_g \approx 0.3$ msec. Although the structure of the formulas defining the time of the flow of the channel τ in response to capillary and gravitational forces are different, the numerical estimates for typical cases of narrow channels show that Eqs. (25) and (28) give values of the same order.

The following relation gives approximately the resultant filling time when both forces act together:

$$\tau^{-1} \cong \tau_g^{-1} + \tau_\sigma^{-1} \tag{29}$$

This analysis shows that the difference in the collapse models does not lead to a substantial quantitative difference.

We now consider the filling of a narrow channel by which we mean one whose diameter does not exceed the capillary constant $a_k = (2\sigma/\rho g)^{1/2}$, in particular $a_k = 0.122$ cm for water, while $a_k \sim 0.5$ cm for molten metals, the filling being in response to gravity and pressure gradient (Fig. 2). We neglect the closure of the channel in the radial direction due to capillary forces.

The Navier–Stokes equation takes the form

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right) - \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} \right) = -\rho g - \frac{\partial P}{\partial z} \tag{30}$$

We neglect the radial motion, so v_z is not dependent on z, as follows from the equation of continuity, i.e., $v_z = f(t)$; Eq. (30) becomes

$$\rho \frac{\partial v_z}{\partial t} - \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{d v_z}{\partial r} \right) = -\rho g - \frac{\partial P}{\partial z} \tag{31}$$

The channel fills from the bottom as the liquid flows along the walls; the motion is complicated in the regions $z < -H(t)$ and $z > 0$ (Fig. 2), so we consider the motion only for the region $0 > z > -H(t)$, and find the filling time from the condition of conservational matter. We neglect the dimensions of the part $-H(t) + R > z > -H(t)$ as these are small for a narrow channel and neglect the radial motion in the region $0 > z > -H(z)$, with $v_z = v_z(r, t)$ to assume that the pressure within the liquid is not dependent on r. The condition at the free boundary

$$\tau_{zz} = -P + 2\mu \, \partial v_z / \partial z = \sigma / R$$

gives us for $z = -H(t)$ that $P(-H) = -\sigma/R$; similarly we find $P(0) = \sigma/R^*$, where R is the channel radius in the lower part, while R^* is the mean radius of the liquid in the upper part.

We integrate Eq. (31) with respect to z from 0 to $-H$ to get

$$\rho \frac{\partial v_z}{\partial t} - \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right) = -\rho g - \frac{\sigma}{H(t)} \left(\frac{1}{R} + \frac{1}{R^*} \right) \tag{32}$$

Then the capillary forces act in the same sense as the gravitational forces and behave as though they were bulk forces with the pressure gradient

$$\partial P / \partial z = (^1 / R^* + {}^1 / R) \sigma / H(t)$$

The law of conservation of mass [Eq. (4)] takes the form

$$\int_0^{2\pi} \int_R^{R^*} \int_0^t v_z r dr \, d\varphi \, dt = \pi R^2 [H(t) - H_0] \tag{33}$$

so the equations describing the motion of the liquid take the following form:

$$2 \int_R^{R^*} \int_0^t v_z(r, t) r \, dr \, dt = R^2 [H(t) - H_0] \tag{34}$$

$$\rho \frac{\partial v_z}{\partial t} - \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right) = -\rho g \left[1 + \frac{\sigma}{\rho g H(t)} \left(\frac{1}{R^*} + \frac{1}{R} \right) \right] \tag{35}$$

System (34)-(35) defines the law $H = H(t)$, which can be used to find τ from the condition $H = 0$ for $t = \tau$.

Equation (35) has no solution in the general case, but it can be solved in two limiting cases: high and low viscosities.

If Re is small, the main part is played by the viscous term, and we can neglect the inertial term; then Eq. (32) gives

$$\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} = \frac{\rho g}{\mu} \left[1 + \frac{\sigma}{\rho g H} \left(\frac{1}{R^*} + \frac{1}{R} \right) \right] = F_1(t) \tag{36}$$

Then

$$v_z(r, t) = {}^1/_4 (R^{*2} - r^2) F_1(t) \tag{37}$$

We substitute Eq. (37) into (33) and integrate with respect to r to get

$$\int_0^t F_1(t) \, dt = \frac{8R^2}{(R^{*2} - R^2)^2} [H - H(t)] \tag{38}$$

We differentiate Eq. (38) with respect to t and substitute for $F_1(t)$ to get

$$\frac{\rho g}{\mu} \left[1 + \frac{\sigma}{\rho g H} \left(\frac{1}{R^*} + \frac{1}{R} \right) \right] = -\frac{8R^2}{(R^{*2} - R^2)} \frac{dH}{dt} \tag{39}$$

Then

$$\frac{dH}{dt} + \frac{\sigma (R^{*2} - R^2)}{8\mu R^2} \left(\frac{1}{R^*} + \frac{1}{R} \right) \frac{1}{H} + \frac{\rho g (R^{*2} - R^2)^2}{8\mu R^2} = 0$$
$$\text{for} \quad t = 0, \quad H = H_0 \tag{40}$$

The solution to Eq. (40) is

$$\beta t = H_0 - H(t) + \frac{\alpha}{\beta} \ln \frac{\alpha + \beta H(t)}{\alpha + \beta H_0}$$
$$\alpha = \frac{\sigma (R^{*2} - R^2)^2}{8\mu R^2} P^*, \qquad \beta = \frac{\rho g (R^{*2} - R^2)^2}{8\mu R^2} \tag{41}$$
$$P^* = (1/R^* + 1/R), \qquad \alpha/\beta = P^*\sigma/\rho g$$

From Eq. (41) with $t = \tau$, $H(\tau) = 0$ we get

$$\tau = \frac{8\mu R^2}{\rho g (R^{*2} - R^2)^2} \left[H_0 - \frac{\sigma}{\rho g} P^* \ln \left(1 + \frac{\rho g}{\sigma} \frac{H_0}{P^*} \right) \right] \tag{42}$$

The first term in Eq. (42) represents the flow in response to the gravitational forces alone:

$$\tau_g = 8\mu R^2 H_0 / \rho g (R^{*2} - R^2)^2 \tag{43}$$

The second term incorporates the reduction in filling time due to the capillary forces. A numerical estimate can be made. A narrow channel in a molten metal ($\sigma = 10^3$ dyne/cm, $\mu' = 5 \cdot 10^{-2}$ Poise, $\rho = 10$ g/cm^3, $a^2 = \sigma/\rho g = 0.1$ cm^2, $\mu/\rho g = 5 \cdot 10^{-6}$ cm$\cdot$sec) gives $\tau_g \approx 0.1$ msec for $R = 10^{-2}$ cm and $R^* = 0.1$ cm; the viscosity can have a marked effect on the motion only when Re is small, so the characteristic dimension R^* should be small. The total filling time incorporating the capillary forces is $\tau \approx 0.01$ msec, i.e., is less by an order of magnitude. This time has been underestimated, since it does not incorporate the initial acceleration required for the channel filling rate to reach a steady value.

We incorporate this initial stage by leaving the inertial term and neglecting the viscosity; this approximation corresponds to the second limiting case (small viscosity).

The equations for the velocity are

$$\frac{\partial v_z}{\partial t} = g \left[1 + \frac{\sigma}{\rho g H} P^* \right] = F_2(t) \tag{44}$$

$$2 \int_R^{R^*} \int_0^t v_z(r, t)\, r\, dr\, dt = R^2 (H - H_0) \tag{45}$$

We cannot satisfy the boundary condition $v = 0$ at $r = R^*$ if we neglect the viscosity, but the boundary layer will be thin if R^* is sufficiently large, and it can be neglected for small times.

As $v_z(r) = \text{const}$, we have from Eq. (45) that

$$\int_0^t v_z(t)\, dt = \frac{R^2}{(R^{*2} - R^2)} [H_0 - H(t)] \tag{46}$$

$$\frac{\partial v_z}{\partial t} = g \left[1 + \frac{\sigma P^*}{\rho g H(t)} \right] = F_2(t) \tag{47}$$

Then we get the equation for $H(t)$:

$$\frac{d^2 H}{dt^2} + \frac{\alpha_1}{H} + \beta_1 = 0$$
$$\alpha_1 = \frac{\sigma (R^{*2} - R^2)}{\rho R^2} P^*, \qquad \beta_1 = \frac{g (R^{*2} - R^2)}{R^2} \tag{48}$$

The first integral of Eq. (48) takes the form

$$dH/dt = -[2\beta_1 (H_0 - H) - 2\alpha_1 \ln (H/H_0)]^{1/2} \tag{49}$$

Consequently,

$$\tau = \frac{1}{\sqrt{2}} \int_0^{H_0} [\beta_1 (H_0 - H) - \alpha_1 \ln (H / H_0)]^{-1/2} dH \tag{50}$$

Neglecting the accelerating capillary forces we have

$$\tau_g = \frac{1}{\sqrt{2}} \int_0^{H_0} \frac{dH}{\sqrt{\beta_1 (H_0 - H)}} = \sqrt{\frac{2R^2 H_0}{g (R^{*2} - R^2)}} \approx \frac{R}{R^*} \sqrt{\frac{2H_0}{g}} \tag{51}$$

The following are some numerical estimates. Let H = 2.5 cm, R = 10^{-2} cm; then $\tau \cong 0.7$ msec for R^*= 1 cm for a molten metal, and $\tau \simeq 7$ msec for R^* = 0.1 cm; in the general case, Eq. (50) can be put as

$$\tau = \frac{H_0}{\sqrt{2\alpha_1}} \int_0^1 \frac{dU}{\sqrt{\gamma (1 - U) - \ln U}}, \quad \gamma = \frac{\beta_1}{\alpha_1} H_0 = \frac{\rho g}{\sigma P^*} H_0 \tag{52}$$

We find τ_σ neglecting the gravitational forces, in this case Eq. (52) becomes

$$\sqrt{2\alpha_1}\, \tau_\sigma = H_0 \int_0^1 \frac{dU}{\sqrt{-\ln U}} = 2H_0 \int_0^\infty \exp(-v^2)\, dv = H_0 \sqrt{\pi}$$

Then

$$\tau_\sigma = H_0 \sqrt{\frac{\pi \rho R^2}{2\sigma (R^{*2} - R^2) P^*}} \approx H_0 \frac{R}{R^*} \sqrt{\frac{\pi \rho R}{2\sigma}} \tag{53}$$

For H = 2.6 cm, ρ= 10 g/cm^3, σ = 10^3 dyne/cm, R = 10^{-2} cm, R^* = 1 cm we have $\tau_\sigma \approx 0.31$ msec, while $\tau_\sigma \approx 3.1$ msec for R^* =0.1 cm.

Then Eq. (29) gives us the resultant filling time incorporating gravitational and capillary forces; we substitute the numerical values from the above example and get $\tau \approx 0.21$ msec.

In estimating τ_σ we neglected effects related to the boundary layer; the thickness of the boundary layer is $\delta \approx 4\sqrt{\nu t}$ [6]; the numerical values give $\delta \approx 3 \cdot 10^{-3}$ cm, and we have [6] for $(\nu t/R^2) < 5 \cdot 10^{-3}$ that the thickness of this layer is less than 0.1 R, while over the rest of the region the velocity is independent of r, so we can use Eqs. (51)-(53) with reasonable accuracy to estimate the corresponding times.

We now consider how far these qualitative estimates agree with experiment; it is stated [3] that the channel in VKZh-94 oil persists for about $5 \cdot 10^{-2}$ sec in response to a focused electron beam. Our estimates indicate that the channel in the oil persists longer than one in a molten metal on account of the reduced surface tension. Numerical estimates with the corresponding values inserted in the formula for the collapse time do not give satisfactory agreement with the experiments of [3] within an order of magnitude. A possible reason is neglect of the thermal effect accompanying the hydrodynamic ones. The characteristic time for thermal relaxation is $\tau_h \sim R^2/a_t$, where a_t is the thermal diffusivity, and τ_h is about 10^{-3} sec for R = 10^{-2} cm, a_t = 0.1 cm^2 sec, i.e., is of the same order as the collapse time. Then vapor can appear within the channel, which increases the collapse time relative to the purely hydrodynamic case. The increase in collapse time can also be due to energy release in the final stage of the process, which has not been considered here.

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